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# Testing Gravity on Astrophysical Scales

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We summarize a series of observational tests of the law of gravity on large astrophysical scales. These tests account for testing both the Poisson equation (inverse square law) using weak lensing and the Einstein equivalence principle through the test of the constancy of the constants of Nature. We emphasize the need to test general relativity on cosmological scales in light of the cosmological constant problem and of recent observational claims concerning the variation of fine structure constant.

**KEY WORDS:** gravity; astrophysical scales.

## 1. TESTING GRAVITY ON LARGE SCALES

Cosmological observations have led to the "evidence" that the universe is undergoing a late time acceleration (Peebles and Ratra, in press). The interpretation of this conclusion is still a matter of debate. At the lowest level, the conclusion to be reached is that the Friedmann equations for a universe filled only with normal matter (i.e. radiation and dust) cannot explain the current data. There are different ways of facing this fact. Either one can conclude that the interpretation of the cosmological data is not correct (i.e. we do not accept the evidence for the acceleration of the universe, see Peebles and Ratra, in press, for a recent critical review and Buchert and Carfora, 2002, for an interesting proposal) or one tries to introduce new degrees of freedom in the cosmological model at hand. In this latter case, these extra degrees of freedom, often referred to as *dark energy*, can be introduced as a new kind of matter (including a cosmological constant, quintessence, . . .) or as a new property of gravity.

In the first, and most common, approach one keeps general relativity unchanged while introducing new forms of gravitating matter (dark matter to explain galaxy rotation curves and matter with negative pressure to explain the acceleration of the universe) beyond the standard model of particle physics. But, one is still left

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with the cosmological constant problem (Weinberg, 1989) (why is the density of vacuum energy expected from particle physics so small?) unsolved as well as the time coincidence problem (why does the dark energy starts dominating today?).

Another route is to allow for modifications of gravity in the infrared, i.e. at large distances, in order to explain why a large vacuum energy density does not give rise to a large curvature. Such alternatives have recently received more attentions in the framework of braneworld models in which the standard model fields are localized on a three-dimensional brane embedded in a higher dimensional spacetime. All higher dimensional models predict that gravity should depart from its standard Newton behavior on *small scales* and up to now this scale is constrained to be smaller than 100–500 $\mu$m (Long *et al.*, 2002). Among braneworlds models, a class of models have also the feature to allow for deviation from four-dimensional Einstein gravity on large scales. This is for example the case of some multibrane models (Gregory *et al.*, 2000), multigravity (Kogan *et al.*, 2000), brane induced gravity (Dvali *et al.*, 2000) or simulated gravity (Carter *et al.*, 2000; Uzan, 2002, in press). In such models, gravity is not mediated only by a massless graviton (hence breaking one of the hypothesis of Weinberg's theorem (Weinberg, 1989), so that one expects to have deviations from Newton inverse square law on large scales. From a cosmological point of view, it was shown that a common feature of these modifications of gravity was to lead to an accelerated expansion of the universe without introducing matter with negative pressure (Damour *et al.*, 2002a; Deffayet, 2001; Deffayet *et al.*, 2002). It seems that these models also induce measurable deviations from the predictions of general relativity on Solar System scales (Dvali *et al.*, 2002; Lue and Starkmann, 2002). Needless to say that the investigations of the relations between local test of gravity and cosmological behavior are promising. Other phenomenological approaches such including a modification of inertia (Milgrom, 2002) or a nonlocal modification of gravity (Arkani-Hamed *et al.*, 2002) have also been proposed.

From a more theoretical point of view, string theory seems to be the only known promising framework that can reconcile quantum mechanics and gravity, even though it is not yet fully defined beyond the perturbative level. One definitive prediction drawn for the low-energy effective action is the existence of extra-dimensions and of a scalar field, the dilaton, that couples to matter (Taylor and Veneziano, 1988; Witten, 1984) and whose expectation value determines the string coupling constant. It follows that the low-energy coupling constants are in fact dynamical quantities. When the dilaton is massless (or almost) it leads to three effects: (i) a scalar admixture of a scalar component inducing deviations from general relativity in gravitational effects, (ii) a variation of the couplings, and (iii) a violation of the weak equivalence principle. If the dilaton were to remain massless, it would induce a violation of the universality of free fall seven order of magnitudes larger than the actual bounds (Damour and Polyakov, 1994a,b). In that framework, the question would actually be "why are the constants so constant?"

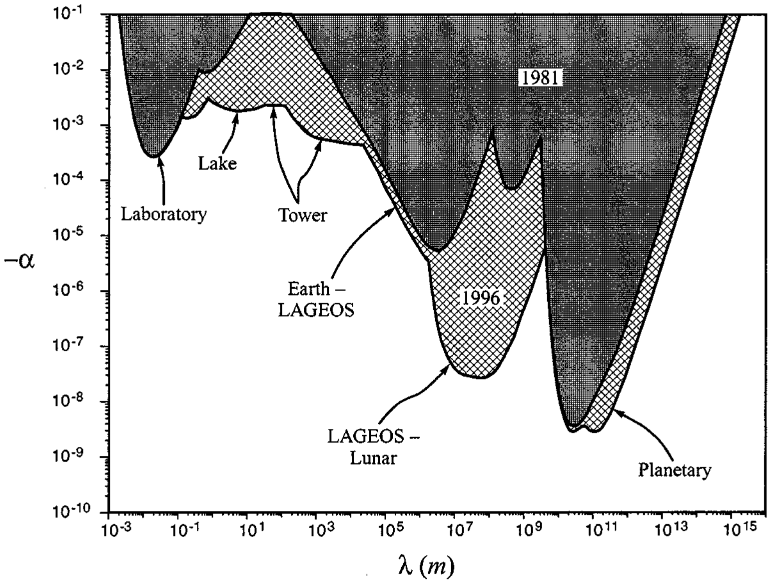


**Fig. 1.** Summary of the constraints on a Yukawa type deviation to the Newton's law on Solar System scales. The gravitational potential is parameterized by $V = (1 + \alpha e^{-r/\lambda})$ $V_{\rm Newton}$. From Fischbach and Tamadge (1999; Chen and Cook, 1993).

To avoid such a catastrophe, it has either to suddenly take a mass larger than a few meV (so that gravity will be compatible with Einstein gravity above a millimeter) or decouple from matter (Damour and Polyakov, 1994a,b). Both mechanisms have different implication concerning the variation of the coupling constants. As a conclusion, testing for the constancy of constants may reveal the existence of further gravitational fields or of extra-dimensions and it opens an observational window on the low-energy limit of string theory and on the stabilization of the dilaton and extra-dimensions.

All these considerations are driving to develop new tests of gravity on cosmological scales that will probe both the inverse square law behavior and the Einstein equivalence principle. Up to now, the observational status is the following.

1. On Solar System size, the Newton law as well as the universality of free fall are tested with a very good accuracy (see Fig. 1 for a summary of the constraints on a Yukawa type fifth force and Will, 1993, 2001). One question mark may however be the Pioneer results (Anderson *et al.*, 1998, 2002), if it turns out to be confirmed.
2. On galactic scales, there are a number of astrophysical constraints that a successful modification of gravity will have to face (see e.g. Aguirre

*et al.*, 2001, where most of the constraints have been discussed in details), specially if the considered modification aims at explaining the galaxy rotation curves. Note that this is in general not the case of the models trying to explain the acceleration of the universe by a modification of gravity on large scales. It is not clear yet that a unique model will be able to explain both the flat rotation curves and the acceleration of the universe, as it is not clear that a unified model of dark matter-energy exists. If the modification of gravity has some relevance on galactic scales then it will have to explain the flatening of the rotation curves and to account for the dependence of the galaxy rotation curve on the luminosity of the galaxy. This dependence is encapsuled in the Tully–Fischer relation relating the luminosity of a spiral galaxy to its assymptotic rotation velocity $L \propto v_\infty^\alpha$ with $\alpha \sim 4$. This sets severe constraints on theories in which the cross-over scale with standard gravity is fixed (see e.g. Sanders, 1986) and favored theories where this cross-over scale depends on the considered galaxy. The compatibility between X-ray and strong lensing observations tends to show that the Poisson equation holds up to roughly 2 Mpc (Allen *et al.*, 2000).

3. On cosmological scales, there is at the moment no direct tests of gravity. Indeed the growth of cosmological structure can put some indirect constraints but usually the observations entangle the properties of the matter and gravity. Recently, it has been claimed that observations of quasars absorption spectra were in favor of a lower fine structure constant in the past. If real, this will be an indication of the breakdown of the Einstein equivalence principle. The acceleration of the universe may be related to such a cosmological variation of the coupling constants, e.g. in quintessence models (Damour *et al.*, 2002b; Wetterich, 2002).

To summarize, general relativity is fairly well tested up to galactic scales but there remain some observational puzzles that still drive to question its validity: the Pioneer effect (if real), the flatening of rotation curves, the acceleration of the universe and the variation of some constants (if confirmed). Among these four puzzles, two are usually solved by adding matter beyond the standard model of particle physics. Testing gravity on astrophysical scales will either enable to build up an alternative (unified?) explanation or validate the standard approach.

In this proceedings, we first recall in Section 2, a proposal the inverse square law on large (astrophysical) scales that I recently proposed with Francis Bernardeau (Uzan and Bernardeau, 2001) (see also White and Kochanek, 2001). Then I review the tests of the constancy of the constant of nature (Section 3) and give a summary of the extensive review (Uzan, in press-b). In particular, I focus on the constraints on the fine structure constant and discuss some recent claims concerning its possible variation.

## 2. TESTING NEWTON'S LAW ON LARGE SCALES

The deflection of light by a gravitational potential, first observed during the Solar eclipse of the 29th May 1919 by the expeditions lead respectively by Eddington and Cottingham in Principia island, and Davidson and Crommelin in the Nordeste of Brazil, was at the heart of the first test of general relativity.

This test checks that the deflection of light by the Sun gravitational field is the one predicted by the theory of general relativity. Indeed, to be conclusive, these experiments have to assume that the mass of the Sun is known. What is really tested is thus the consistency between the mass of the Sun and the gravitational field it creates, i.e. the weak field limit of the Einstein equations. Recently, we proposed that such a test can be extended to astrophysical scales where there is, at the moment, no test of the gravitational law. The bending of light by a matter distribution is intrinsically a relativistic effect which enables to test gravity at extragalactic scales.

To sketch, the effect of a modification of gravity on large scales and the method we proposed, let us assume that the background spacetime can be described by a Friedmann–Lemaître spacetime. As long as we are dealing with subhorizon scales, we can take the metric to be of the form, assuming flat spatial sections,

$$ds^2 = -(1-2\Phi)dt^2 + a^2(1+2\Phi)(d\chi^2 + \chi^2 d\Omega^2) \tag{1}$$

where $t$ is the cosmic time, $a(t)$ the scale factor, $\chi$ the comoving radial coordinate and $d\Omega^2$ the unit solid angle. In a Newtonian theory of gravity, $\Phi$ is the Newtonian potential $\Phi_N$ determined by the Poisson equation

$$\Delta\Phi_N = 4\pi G\rho a^2\delta \tag{2}$$

where $G$ is the Newton constant and $\Delta$ the three-dimensional Laplacian in comoving coordinates, $\rho$ the background energy density and $\delta \equiv \delta\rho/\rho$ is the density contrast. If the Newton law is violated above a given scale $r_s$ then we have to change Eq. (2) and the force between two masses distant of $r$ derives from $\Phi = \Phi_N\, f(r/r_s)$ where $f(x) \to 1$ when $x \ll 1$. For instance, choosing $f(x) = 1/(1+x)$ will describe a gravity which is four-dimensional on small scales and that becomes five-dimensional of large scales. Using Eq. (2) it leads, with $\mathbf{r} = a\mathbf{x}$, to

$$\Phi(\mathbf{x}) = -G\rho a^2 \int d^3\mathbf{x}' \frac{\delta(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\, f\left(\frac{|\mathbf{x}-\mathbf{x}'|}{x_s}\right). \tag{3}$$

If the Poisson equation is satisfied then the power spectra of the density contrast and gravitational potential have to satisfy

$$\mathcal{P}_{\Delta\Phi_N}(k) = (4\pi G\rho a^2)^2 \mathcal{P}_\delta(k), \tag{4}$$

that is

$$\mathcal{P}_{\Delta\Phi}(k) = (4\pi G\rho a^2)^2 \mathcal{P}_\delta(k)\, f_c(kr_s)^2 \tag{5}$$

where $f_c$ can be expressed in terms of the Fourier transform of $f$ (see Fig. 1).

A way to test the validity of the Newton law is thus to test the validity of Eq. (2) which is possible if one can measure $\delta$ and $\Phi$ independently.

From galaxy catalogs, one can extract a measurement of the two-point correlation function of the cosmic density field, $\xi(r) \equiv \langle \delta(0)\delta(\mathbf{r})\rangle$. It leads to a measurement of

$$\mathcal{P}(k) = \frac{1}{(2\pi)^2}\int \xi(r)\,\frac{\sin kr}{kr} r^2\,dr. \tag{6}$$

On the other hand, weak lensing surveys offer a novel and independent window on the large scale structures. Weak lensing measurements are based on the detection of coherent shape distortions of background galaxies due to the large scale gravitational tidal forces. The deformation of a light bundle is characterized by the amplification matrix

$$A_{ab} \equiv \begin{pmatrix} 1-\kappa-\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}. \tag{7}$$

$\vec{\gamma}$, the shear, can be measured from galaxy ellipticities (Bacon *et al.*, 2000; Kaiser *et al.*, 2000; Van Waerbeke *et al.*, 2000; Wittman *et al.*, 2000) from which one can reconstruct the convergence $\kappa$. The convergence is generated by the cumulative effect of large scale structures along the line of sight (Bartelmann and Schneider, 1999; Mellier, 1999). In a direction $\vec{\theta}$ it reads,

$$\kappa(\vec{\theta}) = \int_0^{\chi_S} g(\chi)\Delta_2\Phi(\mathcal{D}(\chi)\vec{\theta},\chi)\,d\chi. \tag{8}$$

where $\mathcal{D}$ is the comoving angular diameter distance and where $\Delta_2$ is the two-dimensional Laplacian in the plane perpendicular to the line of sight. The function $g$ depends on the radial distribution of the sources by

$$g(\chi) = \int_0^{\chi} d\chi' n(\chi')\frac{\mathcal{D}(\chi-\chi')\mathcal{D}(\chi')}{\mathcal{D}(\chi)}. \tag{9}$$

$\kappa(\vec{\theta})$ is a function on the celestial sphere that can be decomposed, in the small angle approximation, in Fourier modes

$$\hat{\kappa}(\mathbf{l}) = \int \frac{d^2\vec{\theta}}{2\pi}\kappa(\vec{\theta})\,e^{i\mathbf{l}.\vec{\theta}} \tag{10}$$

so that, using the expression (8) and the definition of the angular power spectrum of $\kappa$ as $\langle\hat{\kappa}(\mathbf{l})\hat{\kappa}^*(\mathbf{l}')\rangle = (2\pi)^{-1}\mathcal{P}_\kappa(l)\delta^{(2)}(\mathbf{l}-\mathbf{l}')$, we obtain

$$\mathcal{P}_\kappa(l) = \int d\chi\,\frac{g^2(\chi)}{\mathcal{D}^2(\chi)}\mathcal{P}_{\Delta\Phi}\left(\frac{l}{\mathcal{D}(\chi)}\right). \tag{11}$$

It clearly appears that cosmic shear measurements are a direct probe of the gravitational potential. By comparing weak lensing and galaxy catalogs measurements, we have a test of the Poisson equation.

So far cosmic shear has been detected up to a scale of about 2 $h^{-1}$ Mpc (Bacon *et al.*, 2000; Kaiser *et al.*, 2000; Van Waerbeke *et al.*, 2000; Wittman *et al.*, 2000) ($h$ being the Hubble constant in units of 100 km$^{-1}$s$^{-1}$Mpc). This method is in principle applicable to any scale up to 100 $h^{-1}$ Mpc. With galaxy surveys such as SDSS that will measure $\mathcal{P}_\delta$ up to 500 $h^{-1}$ Mpc (Vogeley, 1998) it makes possible comparisons of $\mathcal{P}_\delta$ and $\mathcal{P}_{\Delta\Phi}$ on cosmological scales therefore enabling direct tests of the gravity law up to roughly 100 $h^{-1}$ Mpc.

To illustrate this discrepancy we consider the growth of the perturbations on scales from 10 to some hundreds of Mpc in a modified gravity scenario. For that purpose, we assume that the standard behavior of the scale factor is recovered (i.e. we have the standard Friedmann equations). Note that this is the case in most of the scenarios in which such modifications of gravity occur and in which the evolution of the scale factor is modified in the same way as by a cosmological constant.

In the weak field limit and for a pressureless fluid, taking advantage of the fact that the relation between $\delta$ and $\Phi$ is local in Fourier space, $\delta$ can be shown to satisfy the evolution equation

$$\ddot{\delta}_k - 2H\dot{\delta}_k - \frac{3}{2}H^2\Omega(t)\, f_c\left(k\frac{r_s}{a(t)}\right)\delta_k = 0 \qquad (12)$$

where a dot refers to a derivative with respect to $t$. $H \equiv \dot{a}/a$ is the Hubble parameter.

Looking for a growing mode as $\delta_k \propto t^{\nu+(\kappa)}$ in a Einstein-de Sitter matter dominated universe ($\Omega = 1$, $H = 2/3t$) gives a growing solution such that $\nu_+(k) \to 2/3$ for $kx_s \gg 1$ and $\nu_+(k) \to 0$ for $kx_s \ll 1$. At large scales the fluctuations stop growing mainly because gravity becomes weaker and weaker. In Fig. 2, we depict the numerical integration of the growing mode and the resulting power spectrum assuming that $f(x) = 1/(1+x)$. Note that since $x_s$ and the comoving horizon respectively scale as $a^{-1}$ and $\sqrt{a}$ (in an Einstein-de Sitter universe) $x_s$ enters the horizon at about 760 $h^{-1}$ Mpc if $r_s = 50\,h^{-1}$ Mpc. Thus, all the modes with comoving wavelengths smaller than 760 $h^{-1}$ Mpc feel the modified law of gravity only when they are subhorizon. As a consequence, it is well justified for all the observable modes (i.e. up to 500 $h^{-1}$ Mpc) to consider the effect of the non-Newtonian gravity in the subhorizon regime only.

Let us emphasize that, in Fig. 2, the deviation from the standard behavior of the matter power spectrum is model dependent (it depends in particular on the cosmological parameters), but that the discrepancy between the matter and gravitational potential Laplacian power spectra is a direct signature of a modified law of gravity. Note that biasing mechanisms (i.e. the fact that galaxies do not necessarily trace faithfully the matter field) cannot be a way to evade this test

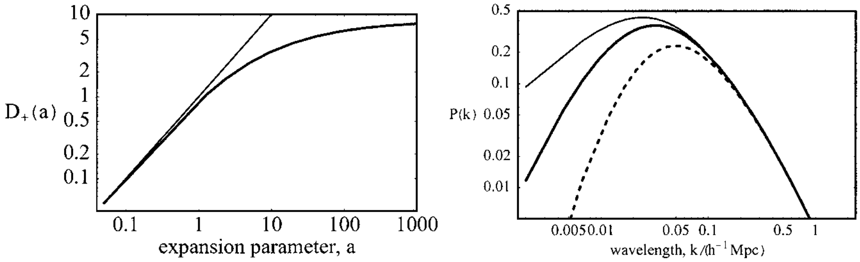


**Fig. 2.** In a theory in which gravity switches from a standard four-dimensional gravity to a five-dimensional gravity above a crossover scale of $r_s = 50\ h^{-1}$ Mpc, there are different cosmological implications concerning the growth of cosmological perturbations. Since gravity becomes weaker on large scales, fluctuations stop growing [left panel] [right panel]. It implies that the density contrast power spectrum (thick line) differs from the standard one (thin line) but, more important, from the gravitational potential power spectrum (dash line).

since bias has been found to have no significant scale dependence at such scales (Narayanan *et al.*, 2000).

## 3. TESTING GRAVITY WITH THE CONSTANTS OF NATURE

At the heart of general relativity is the Einstein equivalence principle that states that (i) the weak equivalence principle (also referred to as the universality of free fall) is valid, i.e. that any electrically neutral test body with negligible gravitational self-energy falls identically, independently of its mass and chemical composition, (ii) the local Lorentz invariance holds, i.e. that the result of any nongravitational experiment is independent of the freely falling referential in which it is performed, and (iii) the local position invariance also holds, i.e. that the result of any nongravitational experiment is independent of where and when it is performed.

If the Einstein equivalence principle is valid then gravity can be described as the consequence of a curved spacetime and is a metric theory of gravity, an example of which are general relativity and the Brans–Dicke theory. This statement is not a "theorem" but there are a lot of indications to back it up (Will, 1993, 2001). Note that superstring theory violates the Einstein equivalence principle since it introduces additional fields (e.g. dilaton, moduli . . .) that have gravitational-strength couplings which violate of the weak equivalence principle. A time variation of a fundamental constant is in contradiction with Einstein equivalence principle since it violates the local position invariance. All new interactions that appear in the extension of standard physics implies extra scalar or vector fields and thus an expected violation of the weak equivalence principle, the only exception being metric theories such as the class of tensor–scalar theories of gravitation in which the dilaton couples universally to all fields and in which one can have a time variation of gravitational constant without a violation of the weak equivalence principle.

By constraining the variation of the fundamental constant on astrophysical scales, we thus test a central hypothesis on which general relativity is built.

### 3.1. Constants That May Vary

The question of the constancy of the constants of physics was probably first addressed by Dirac (1937, 1938, 1974) who expressed, in his "Large Numbers hypothesis," the opinion that very large (or small) dimensionless universal constants cannot be pure mathematical numbers and must not occur in the basic laws of physics. He suggested, on the basis of this numerological principle, that these large numbers should rather be considered as variable parameters characterizing the state of the universe. Dirac noticed a series of numerical coincidences such as between the relative magnitude of electrostatic and gravitational forces between a proton and an electron and[4] $H_0e^2/m_ec^2$ representing the age of the universe in atomic time. He concluded that these coincidences can be "explained" if one assumes that the gravitational constant, $G$, varies with time and scales as the inverse of the cosmic time.

Indeed, there was no theory backing-up this hypothesis but let us stress some interesting points. First, it was argued, using the "anthropic principle," that the coincidences found by Dirac can be derived from physical models of stars and the competition between the weakness of gravity with respect to nuclear fusion (Carter, 1974; Dicke, 1961). Second, let us note that considering a variable constant accounts to considering that it is a dynamical field. This was first pointed out by Jordan (1937, 1939) who proposed the first implementation of Dirac's idea into a field-theory framework. Nevertheless, Dirac's idea motivated many studies concerning the variation of the constant and it was trying, as we also do today, to make a link between the macroscopic and microscopic world and it was mainly driven by the still nonunderstood fact that gravity is very weak compared with other forces of nature.

How many constants should be tested? To be pragmatic, the constancy of all the parameters that are not determined by the theory at hand have to be tested. As an example, the minimal standard model of particle physics plus gravitation that describes the four known interactions depends on 20 free parameters. Either these parameters are not fundamental constants and will be considered as fields in a more general theory, one output of which must be the determination of these parameters, or they are fundamental constants in which case one will only be able to measure them. In that sense, the study of the constants offers a window on the limits of the theory itself.

To finish this series of comments, let us stress that the introduction of constants in physical laws is closely related to the existence of systems of units. It implies

[4] $H_0$ is the value of the Hubble constant today, $e$ the charge of the electron, $m_e$ its mass and $c$ the speed of light.

that the numerical constants is deeply related to the definition of what we call a second, a meter etc ... Since the definition of a system of units and the value of the fundamental constants (and thus the status of their constancy) are entangled, and since the measurement of any dimensionful quantity is in fact the measurement of a ratio to standards chosen as units, *it only makes sense to consider the variation of dimensionless ratios*. We will thus focus on the variation of dimensionless ratios which, for instance, characterize the relative magnitude of two forces, and are independent of the choice of the system of units and of the choice of standard rulers or clocks.

Since we can only measure the variation of dimensionless quantities (such as the ratio of two wavelengths, two decay rates, two cross sections ...), the idea is to pick up a physical system which depends strongly on the value of a set of constants so that a small variation will have dramatic effects. The general strategy is thus to constrain the spacetime variation of an observable quantity as precisely as possible and then to relate it to a set of fundamental constants. This latter step involves limitations related to our theoretical understanding of the considered system.

### 3.2. It Is a Test of the Theory of Gravity

Testing for the constancy of the constants is a test of the Einstein equivalence principle. But let us also stress that if constants are varying one expects also in to have an anomalous force (a "fifth force") which is likely to violate the universality of free fall. Note that to violate the universality of free fall, it requires that the extra-field mediating the new force does not couple universally to all matter fields.

Since the mass of any test body depends on the masses of its constituants and of its binding energy, we expect it to depend on the value of all the coupling constants as well as of the mass of all fundamental particles. This has a profound consequence concerning the motion of any test body.

Let $\alpha$ be any fundamental constant, assumed to be a scalar function and having a time variation of cosmological origin so that in the privileged cosmological rest-frame it is given by $\alpha(t)$. A body of mass $m$ moving at velocity $\vec{v}$ will experience an anomalous acceleration

$$\delta\vec{a} \equiv \frac{1}{m}\frac{dm\vec{v}}{dt} - \frac{d\vec{v}}{dt} = \frac{\partial \ln m}{\partial\alpha}\dot{\alpha}\vec{v}. \tag{13}$$

Now, in the rest-frame of the body, $\alpha$ has a spatial dependence $\alpha[(t' + \vec{v}.\vec{r}/c^2)/\sqrt{1 - v^2/c^2}]$ so that, as long as $v \ll c$, $\nabla\alpha = (\dot{\alpha}/c^2)\vec{v}$. The anomalous acceleration can thus be rewritten as

$$\delta\vec{a} = -\left(\frac{\alpha}{m}\frac{\delta mc^2}{\delta\alpha}\right)\nabla \ln\alpha. \tag{14}$$

The violation of the universality of free fall is quantified by considering the parameter $\eta_{12}$ characterizing the difference of acceleration of two bodies, labelled 1 and 2, and defined by

$$\eta_{12} = 2\frac{|\vec{a}_1 - \vec{a}_2|}{|\vec{a}_1 + \vec{a}_2|}. \tag{15}$$

For two test bodies falling in an external gravitational field, $\vec{g}_{\rm ext}$, one gets a violation of order

$$\eta_{12} = \frac{\partial \ln(m_1/m_2)}{\partial \ln \alpha} c^2 \frac{|\nabla \ln \alpha|}{|\vec{g}_{\rm ext}|}. \tag{16}$$

This anomalous acceleration is generated by the change in the (electromagnetic, gravitational, ...) binding energy (Dicke, 1964, 1969; Haugan, 1979; Nordtvedt, 1990). Besides, the $\alpha$-dependence is a priori composition-dependent. As a consequence, any variation of the fundamental constants will entail a violation of the universality of free fall: the total mass of the body being space dependent, an anomalous force appears if energy is to be conserved.

In conclusion, deviation from Einstein gravity, violation of the universality of free fall and variation of the constants are in general related and expected together so that testing for the constancy of the constants on astrophysical scales opens a window on gravity at these scales.

## 3.3. The Fine Structure Constant: Status of the Constraints

Recent astrophysical observations have restarted the debate concerning the constancy of the fine structure constant, $\alpha_{\rm EM}$. We briefly review in this section the various constraints obtained up to now in order to present the different methods. All the details concerning the different methods and bounds were clearly explained in the literature (Uzan, in press-b).

### *3.3.1. The Oklo Phenomenon*

Oklo is a prehistoric natural fission reactor that operated about $2 \times 10^9$ years ago (corresponding to a redshift of $\sim$0.14) during a few million years in the Oklo uranium mine in Gabon. Two billion years ago, uranium was naturally enriched (because of the difference of decay rate between $^{235}$U and $^{238}$U) and $^{235}$U represented about 3.68% of the total uranium (compared with 0.72% today). Besides, in Oklo the concentration of neutron absorbers which prevent the neutrons from being available for the chain fission was low; water played the role of moderator and slowed down fast neutrons so that they can interact with other $^{235}$U and the reactor was large enough so that the neutrons did not escape faster than they were produced.

From isotopic abundances of the yields, one can extract informations about the nuclear reactions at the time the reactor operated and reconstruct the reaction rates at that time. One of the key quantity measured is the ratio ${}^{149}{}_{62}$SM/${}^{147}{}_{62}$SM of two light isotopes of samarium which are not fission products. This ratio of order of 0.9 in normal samarium, is about 0.02 in Oklo ores. This low value is interpreted by the depletion of ${}^{149}{}_{62}$Sm by thermal neutrons to which it was exposed while the reactor was active.

Shlyakhter (1976) pointed out that the capture cross section of thermal neutron by ${}^{149}_{62}$Sm

$$n + {}^{149}_{62}\text{Sm} \longrightarrow {}^{150}_{62}\text{Sm} + \gamma \tag{17}$$

is dominated by a capture resonance of a neutron of energy of about 0.1 eV. The existence of this resonance is a consequence of an almost cancellation between the electromagnetic repulsive force and the strong interaction. To obtain a constraint, one first needs to measure the neutron capture cross-section of ${}^{149}_{62}$Sm at the time of the reaction and to relate it to the energy of the resonance. One has finally to translate the constraint on the variation of this energy on a constraint on the time variation of the considered constant.

Without going into the details, the key point is the sensitivity of the value of the energy of the resonance, $E_r$ to a change of the fine structure constant that was estimated to be of order (Damour and Dyson, 1996)

$$\alpha_{\text{EM}} \frac{\delta E_r}{\delta \alpha_{\text{EM}}} \sim -1 \text{ Mev.} \tag{18}$$

This large amplification between the resonance energy ($\sim$0.1 eV) and the sensitivity ($\sim$1 MeV) allows to set a constraint of magnitude $|\Delta\alpha_{\text{EM}}/\alpha_{\text{EM}}| \leq E_r/1$ Mev $\sim 10^{-7}$. More precisely, Damour and Dyson (1996) obtained the constraint

$$-0.9 \times 10^{-7} < \Delta\alpha_{\text{EM}}/\alpha_{\text{EM}} < 1.2 \times 10^{-7} \tag{19}$$

at $2\sigma$ level, corresponding to the range $-6.7 \times 10^{-17}$ year$^{-1}$ $< \dot{\alpha}_{\text{EM}}/\alpha_{\text{EM}} < 5.0 \times 10^{-17}$ year$^{-1}$ if $\dot{\alpha}_{\text{EM}}$ is assumed constant.

### *3.3.2 Other Nuclear Constraints*

$\alpha$-, $\beta$-decays and spontaneous fission were also used to constrain the variation of the fine structure constant. The main idea is to extract the $\alpha_{\text{EM}}$-dependence of the decay rate and to use geological samples to bound its time variation.

Many nuclei were used. The sharpest constraint was obtained from the $\beta$-decay of osmium to rhenium by electron emission

$${}^{187}_{75}\text{Re} \longrightarrow {}^{187}_{76}\text{Os} + \bar{\nu}_e + e^-, \tag{20}$$

first considered by Peebles and Dicke (1962). As long as long-lived isotopes are concerned, for which the decay energy $\Delta E$ is small, we can use a nonrelativistic approximation for the decay rate

$$\lambda = \Lambda(\Delta E)^p \tag{21}$$

so that the sensitivity is given by

$$s \equiv \frac{d \ln \lambda}{d \ln \alpha_{\rm EM}} = p \frac{d \ln \Delta E}{d \ln \alpha_{\rm EM}}. \tag{22}$$

Peebles and Dicke (1962) noted that the very small value of its decay energy $\Delta E \simeq 2.5$ keV makes it a very sensitive indicator of the variation of $\alpha_{\rm EM}$. In that case $p \simeq 2.8$ so that $s \simeq -18{,}000$. It follows that a change of about $10^{-2}$% of $\alpha_{\rm EM}$ will induce a change in the decay energy of order of the keV, that is of the order of the decay energy itself.

The data concerning osnium were recenlty updated to take into account the improvements. in the analysis of the meteorite data which now show that the half-life has not varied by more than 0.5% in the past 4.6 Gyr (i.e. a redshift of about 0.45). This implies that (Olive *et al.*, 2002)

$$|\Delta\alpha_{\rm EM}/\alpha_{\rm EM}| < 3 \times 10^{-7}. \tag{23}$$

### *3.3.3. Atomic Spectra*

The previous bounds on the fine structure constant assume that other constants like the Fermi constant do not vary. The use of atomic spectra may offer cleaner tests since we expect them to depend mainly on combinations[5] of $\alpha_{\rm EM}$, $\mu$ and $g_{\rm p}$. Two approaches have mainly received attention: the comparison of atomic clocks in laboratory experiments and the use of quasar absorption spectra.

*Laboratory experiments* are based on the comparison either of different atomic clocks or of atomic clock with ultra-stable oscillators. They are thus based only on the quantum mechanical theory of the atomic spectra. They also have the advantage to be more reliable and reproducible, thus allowing a better control of the systematics and a better statistics. Their evident drawback is their short time scales, fixed by the fractional stability of the least precise standards. This time scale is of order of a month to a year so that the obtained constraints are restricted to the instantaneous variation today, but it can be compensated by the extreme sensibility. They involve the comparison of either ultra-stable oscillators to different composition or of atomic clocks with different species. Solid resonators, electronic, fine structure and hyperfine structure transitions respectively give access to[6] $R_\infty/\alpha_{\rm EM}$, $R_\infty$, $R_\infty\alpha_{\rm EM}^2$, and $g_{\rm p}\mu R_\infty\alpha_{\rm EM}^2$.

Among all experiments, the sharpest constraint has been obtained by Sortais *et al.* (2001) who compared a rubidium to a cesium clock over a period of

[5] $\mu$ is the ratio between the masses of the electron and proton and $g_{\rm p}$ the proton gyromagnetic factor.
[6] $R_\infty$ is the Rydberg constant.

24 months. They deduced that $d\ln(\nu_{\rm Rb}/V_{\rm Cs})/dt = (1.9 \pm 3.1)\times 10^{-15}$ year$^{-1}$. Assuming $g_{\rm p}$ constant, they deduced

$$\dot{\alpha}_{\rm EM}/\alpha_{\rm EM} = (4.2 \pm 6.9)\times 10^{-15}\ \text{year}^{-1}. \tag{24}$$

*The observation of spectra of distant astrophysical objects* encodes information about the atomic energy levels at the position and time of emission. As long as one sticks to the nonrelativistic approximation, the atomic transition energies are proportional to the Rydberg energy and all transitions have the same $\alpha_{\rm EM}$-dependence, so that the variation will affect all the wavelengths by the same factor. Such a uniform shift of the spectra can not be distinguished from a Doppler effect due to the motion of the source or to the gravitational field where it sits. The idea is to compare different absorption lines from different species or equivalently the redshift associated with them.

While performing this kind of observations a number of problems and systematic effects have to be taken into account and controlled: (i) errors in the determination of laboratory wavelengths to which the observations are compared; (ii) while comparing wavelengths from different atoms one has to take into account that they may be located in different regions of the cloud with different velocities and hence with different Doppler redshift; (iii) one has to ensure that there is no light blending; (iv) the differential isotopic saturation has to be controlled. Usually quasars absorption systems are expected to have lower heavy element abundances; (v) hyperfine splitting can induce a saturation similar to isotopic abundances; (vi) the variation of the velocity of the Earth during the integration of a quasar spectrum can induce differential Doppler shift; (vii) atmospheric dispersion across the spectral direction of the spectrograph slit can stretch the spectrum; (viii) the presence of a magnetic field will shift the energy levels by Zeeman effect; (ix) temperature variations during the observation will change the air refractive index in the spectrograph; (x) instrumental effects such as variations of the intrinsic instrument profile have to be controlled. Most of these effects are discussed in the literature (Murphy *et al.*, 2002).

An efficient and extensively used method is to observe fine-structure doublets for which the frequency splitting between the two lines of the doublet is

$$\Delta\nu = \frac{\alpha_{\rm EM}^2 Z^4 R_\infty}{2n^3}\,\text{cm}^{-1}. \tag{25}$$

It follows that $\Delta\nu/\bar{\nu} \propto \alpha_{\rm EM}^2$. It can be inverted to give $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$ as a function of $\Delta\lambda$ and $\bar{\lambda}$ as

$$\left(\frac{\Delta\alpha_{\rm EM}}{\alpha_{\rm EM}}\right)(z) = \frac{1}{2}\left[\left(\frac{\Delta\lambda}{\bar{\lambda}}\right)_z \Big/ \left(\frac{\Delta\lambda}{\bar{\lambda}}\right)_0 - 1\right]. \tag{26}$$

Murphy *et al.* (2001) studied the doublet lines of Si IV, C IV, and Ng II and focused on the fine-structure doublet of Si IV toward eight quasars with redshift $z \sim 2-3$

to get

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (-0.5 \pm 1.3) \times 10^{-5}. \tag{27}$$

Recently Webb *et al.* (1999) introduced a new method referred to as the *many multiplet* method in which one correlates the shift of the absorption lines of a set of multiplets of different ions. One advantage is that the correlation between different lines allows to reduce the systematics. An improvement also arises from the comparison of transitions from different ground-states for ions with very different atomic mass; this increases the sensitivity because the difference between ground-states relativistic corrections can be very large and even of opposite sign.

Webb *et al.* (2001) reanalyzed their initial sample (Web *et al.*, 1999) and included new optical QSO data to have 28 absorption systems with redshift $z =$ 0.5–1.8 plus 18 damped Lyman-$\alpha$ absorption systems towards 13 QSO plus 21 Si IV absorption systems toward 13 QSO. The analysis used mainly the multiplets of Ni II, Cr II, Zn II, and Mg I, Mg II, Al II, Al III, and Fe II were also included. The data were reduced to get 72 individual estimates of $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$ spanning a large range of redshift. From the Fe II and Mg II sample they obtained

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (-0.7 \pm 0.23) \times 10^{-5} \tag{28}$$

for $z = 0.5$–$1.8$ and from the Ni II, Cr II, and Zn II they got

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (-0.76 \pm 0.28) \times 10^{-5} \tag{29}$$

for $z = 1.8$–$3.5$ at a $4\sigma$ level. It refers only to the statistical confidence level. The fine-structure of Si IV gave

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (-0.5 \pm 1.3) \times 10^{-5} \tag{30}$$

for $z = 2$–$3$. These results are summarized in Fig. 3. This series of results is of great importance since all other constraints are just upper bounds. Such a nonzero detection, if confirmed, will have tremendous implications concerning our understanding of physics. Among the first questions that arise, it is interesting to test whether this variation is compatible with other bounds (e.g. test of the universality of free fall), to study the level of detection needed by the other experiments knowing the level of variation by Webb *et al.* (2001), to sort out the amplitude of the variation of the other constants and to ensure that no systematic effects has been forgotten.

### *3.3.4. Cosmological Constraints*

On cosmological scales, the results of primordial nucleosynthesis and the observation of the cosmic microwave background were used to constrain the variation of the fine structure constant.

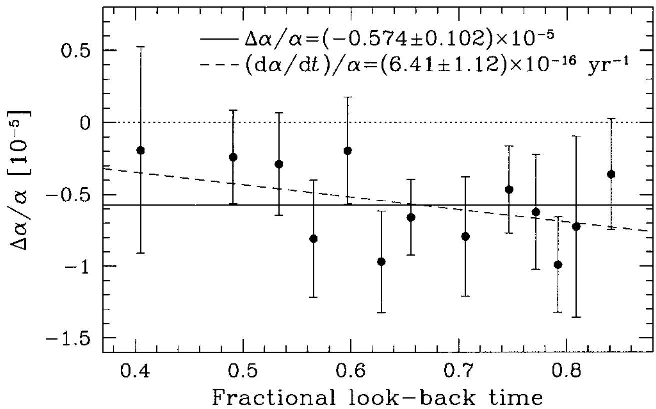


**Fig. 3.** Data points concerning the value of the fine structure constant inferred from the observations of quasar spectra by Webb *et al.* (2001). The best fit of the data of figure, assuming a constant fine structure constant (solid line), does not seem to favor today's value of the fine structure constant (dotted line). This could indicate an unknown systematic effect. Besides, if the variation of $\alpha_{\rm EM}$ is linear (dash line) then these observations are incompatible with the Oklo results. From Murphy *et al.* (2002).

The Cosmic Microwave Background Radiation (CMBR) is composed of the photons emitted at the time of the recombination of hydrogen and helium when the universe was about 300,000 years old. This radiation is observed to be a black body with a temperature $T = 2.723$ K with small anisotropies of order of the $\mu$K. Prior to recombination, the photons are tightly coupled to the electrons, after recombination they can be considered mainly as free particles. Changing the fine structure constant modifies the strength of the electromagnetic interaction and thus the only effect on CMB anisotropies arises from the change in the differential optical depth of photons due to the Thomson scattering, $\dot{\tau} = x_{\rm e} n_{\rm e} c \sigma_{\rm T}$, which enters in the collision term of the Boltzmann equation describing the evolution of the photon distribution function and where $x_{\rm e}$ is the ionization fraction (i.e. the number density of free electrons with respect to their total number density $n_{\rm e}$). The first dependence of the optical depth on the fine structure constant arises from the Thomson scattering cross-section given by

$$\sigma_{\rm T} = \frac{8\pi}{3}\frac{\hbar^2}{m_{\rm e}^2 c^2}\alpha_{\rm EM}^2 \tag{31}$$

and the scattering by free protons can be neglected since $m_{\rm e}/m_{\rm p} \sim 5 \times 10^{-4}$. The second, and more subtle dependence, comes from the ionization fraction.

Avelino *et al.* (2000) claim that BOOMERanG and MAXIMA data favor a value of $\alpha_{\rm EM}$ smaller by a few percents in the past and Battye *et al.* (2001) showed that the fit to current CMB data are improved by allowing $\Delta\alpha_{\rm EM} \neq 0$ and pointed out that the evidence of a variation of the fine structure constant can be thought of as favoring a delayed recombination model. Avelino *et al.* (2001) then performed a joint analysis of nucleosynthesis and CMB data and did not find any evidence for a variation of $\alpha_{\rm EM}$ at one-sigma level at either epoch. They consider the baryon fraction and the fine structure constant as independent and the marginalization over one of the two parameters lead to

$$-0.09 < \Delta\alpha_{\rm EM}/\alpha_{\rm EM} < 0.02 \tag{32}$$

at 68% confidence level. Landau *et al.* (2001) concluded from the study of BOOMERanG, MAXIMA, and COBE data in spatially flat models with adiabatic primordial fluctuations that, at $2\sigma$ level,

$$-0.14 < \Delta\alpha_{\rm EM}/\alpha_{\rm EM} < 0.03. \tag{33}$$

All these works assume that only $\alpha_{\rm EM}$ is varying but one has to assume the constancy of the electron mass. The variation of the gravitational constant can also have similar effects on the CMB (Riazuelo and Uzan, 2002). In conclusion, strong constraints on the variation of $\alpha_{\rm EM}$ can be obtained from the CMB only if the cosmological parameters are independently known.

The amount of $^4$He produced during the big bang nucleosynthesis is mainly determined by the neutron to proton ratio at the freeze-out of the weak interactions that interconvert neutrons and protons. The result of Big Bang nucleosynthesis (BBN) thus depends on $G, \alpha_{\rm EM}$ the weak and strong coupling constants respectively through the expansion rate, the neutron to proton ratio, the neutron–proton mass difference and the nuclear reaction rates, besides the standard parameters such as e.g. the number of neutrino families. In more details, the abundance of $^4$He by mass, $Y_{\rm p}$, is well estimated by

$$Y_{\rm p} \simeq 2\frac{(n/p)_{\rm f}\ \exp(-t_{\rm N}/\tau_{\rm n})}{1+(n/p)_{\rm f}\ \exp(-t_{\rm N}/\tau_{\rm n})} \tag{34}$$

where $(n/p)f = \exp(-Q/kT_{\rm f})$ is the neutron to proton ratio at the freeze-out time, $T_{\rm f}$, and with $Q = m_{\rm n} - m_{\rm p}$. The time $t_{\rm N}$ is the time after which the photon density becomes low enough for the photodissociation to be negligible; it is roughly given by $t_{\rm N} \propto G^{-1/2}T_{\rm N}^{-2}$. $\tau_{\rm n}^{-1} = 1.636 G_{\rm F}^2(1+3g_A^2)m_e^5/(2\pi^3)$ is the neutron lifetime, with $g_A \simeq 1.26$ being the axial/vector coupling of the nucleon. Assuming that the deuteron binding energy is proportional to the square of the strong interaction structure constant, $E_{\rm D} \propto \alpha_{\rm s}^2$ this gives a dependence $t_{\rm N}/\tau_{\rm p} \propto G^{-1/2}\alpha_{\rm s}^2 G_{\rm F}^2$, $G_{\rm F}$ being the Fermi constant.

The light element abundances are thus sensible to the freeze-out temperature, which depends on the Fermi constant, $G$, on the proton–neutron mass difference $Q$, and on the values of the binding energies $B_A$ so that they mainly depend on the four coupling constants and the mass of the quarks. An increase in $G$ or in the number of ultra-relativistic particles results in a higher expansion rate and thus to an earlier freeze-out, i.e. a higher $T_f$. A decrease of the Fermi constant, corresponding to a longer neutron lifetime, leads to a decrease of the weak interaction rates and also results in a higher $T_f$. This shows that primordial nucleosynthesis involves many constants which makes the interpretation of the results more difficult.

Bergström *et al.* (1999) studied the dependence of the thermonuclear rates on $\alpha_{EM}$. Keeping all other constants fixed, assuming no exotic effects and taking a lifetime of 886.7 s for the neutron, it was deduced that

$$|\Delta\alpha_{EM}/\alpha_{EM}| < 2 \times 10^{-2}. \tag{35}$$

### *3.3.5. Universality of Free Fall*

As explained in Section 3.2, one expects to observe a violation of free fall. The most accurate constraints on $\eta_{12}$ are $\eta_{12} = (-1.9 \pm 2.5) \times 10^{-12}$ between beryllium and copper (Su *et al.*, 1994) and $|\eta_{12}| < 5.5 \times 10^{-13}$ between Earth-core-like and Moon-mantle-like materials (Baessler *et al.*, 1999). The Lunar Laser Ranging (LLR) experiment gives the bound $\eta_{12} = (3.6 \pm 4) \times 10^{-13}$ (Müller *et al.*, 1999).

The LLR constraint, $|\vec{a}_{\rm Earth} - \vec{a}_{\rm Moon}| \leq 10^{-14}$ cms$^{-2}$, implies that on the size of the Earth orbit

$$|\nabla \ln \alpha_{EM}| \leq 10^{-33} - 10^{-32}\ \mathrm{cm}^{-1}. \tag{36}$$

Extending this bound to the Hubble size leads to the estimate $\Delta\alpha_{EM}/\alpha_{EM} \leq 10^{-4}$–$10^{-5}$. This indicates that if the claim by Webb *et al.* (2001) is correct then it should induce a detectable violation of the equivalence principle by coming experiments such as MICROSCOPE[7] and STEP.[8] They will respectively have an accuracy of the level $\eta \sim 10^{-15}$ and $\eta \sim 10^{-18}$.

## 3.4. Other Constants

The variation of other constants have also been considered. This is the case of the gravitational constant, the electron to proton mass ratio, the weak and strong interaction structure constants and the proton gyromagnetic factor. A description of the observations used in these cases can be found in the literature (Uzan, in press-b), as well as an up to date list of the observational constraints. Let us just discuss the latest results concerning the electron to proton mass ratio, $\mu$.

More recently, Ivanchik *et al.* (2001) measured, with the VLT, the vibro-rotational lines of molecular hydrogen for two quasars with damped Lyman-$\alpha$ systems respectively at $z = 2.3377$ and 3.0249 and also argued for the detection

[7] http://sci2.esa.int/Microscope/
[8] http://einstein.stanford.edu/STEP/

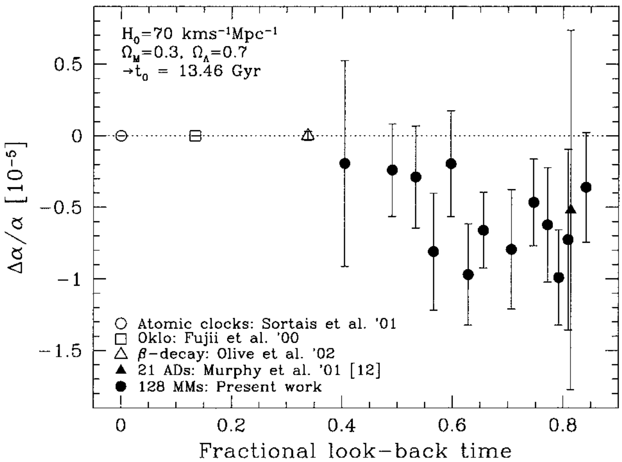


**Fig. 4.** Summary of the different constraints described in the text. From Murphy *et al.* (2002).

of a time variation of $\mu$. Their most conservative result is (the observational data were compared to two experimental data sets)

$$\Delta\mu/\mu = (-5.7 \pm 3.8) \times 10^{-5} \qquad (37)$$

at $1.5\sigma$ and the authors cautiously point out that additional measurements are necessary to ascertain this conclusion.

## 4. WHAT SHOULD WE CONCLUDE?

The constraints concerning the fine structure constant are summarized in Fig. 4. The nonzero detections by Webb *et al.* (2001) draw the questions of their compatibility with the bounds obtained from other physical systems but also, on a more theoretical aspect, of the understanding of such a late time variation which does not seem to be natural from a field theory point of view.

Theoretically, one expects *all* constants to vary (e.g. in GUT, Kaluza–Klein and string inspired models) and the levels of their variation are correlated. Better analysis of the degeneracies are really needed before drawing definitive conclusions but such analysis are also dependent in the progresses in our understanding of the fundamental interactions and particularly of the QCD theory and on the generation of the fermion masses.

In a given theoretical framework, one can deduce the relation between the variation of different constants as well as their time behavior. Concerning the time variation, the result by Webb *et al.* (2001) is not compatible with the Oklo or Re/Os results if the variation is linear with time (see Fig. 3). The recent bound by Olive *et al.* (2002) at $z \sim 0.45$ emphasizes the difficulty to achieve such a late time deviation and shows that the stabilization of the fine structure constant, if it has varied, had to occur very quickly. Concerning the compatibility with other measurements, it can be concluded that, in a GUT framework, the result by Webb *et al.* (2001) is neither compatible with the constraints on the variation of $\mu$ and $g_p\alpha_{EM}^2$ from quasar spectra nor with the bound by Murphy *et al.* (2001) using the Si IV doublet. Both results (on $\alpha_{EM}$ and $\mu$) arise from the observation of quasar absorption spectra; it is of importance to ensure that all systematics are taken into account and are confirmed by independent teams, using e.g. the VLT which offers a better signal to noise and spectral resolution.

The developments of high energy physics theories such as multidimensional and string theories provide new motivations to consider the time variation of the fundamental constants (see Uzan, in press-b, for a review of the theoretical motivations). The observation of the variability of these constants constitutes one of the very few hope to test directly the existence of extra-dimensions. In the long run, it may help to discriminate between different effective potentials for the dilaton and/or the dynamics of the internal space. But indeed, independently of these motivations, the understanding of the value of the fundamental constants of nature and the discussion of their status of constant remains a central question of physics in general: questioning the free parameters of a theory accounts to question the theory itself. The step from the standard model+general relativity to string theory allows for dynamical constants and thus starts to address the question of why the constants have the value they have. Unfortunately, no complete and satisfactory stabilization mechanism is known yet and we have to understand why, if confirmed, the constants are still varying. Remind that the dilaton has to become massive or to decouple for low-energy effective string theory to be compatible with the test of general relativity.

Let us also emphasize an important issue. If the fine structure constant has varied in a close past then, in an effective four-dimensional theory, the only consistent approach to make a Lagrangian parameter time dependent is to consider it as a field $\phi$ say. The Klein–Gordon equation for this field ($\ddot{\phi} + 3H\dot{\phi} + m^2\phi + \cdots = 0$) implies that $\phi$ is damped as $\dot{\phi} \propto a^{-3}$ if its mass is much smaller than the Hubble scale and that it oscillates if its mass is much larger than the Hubble scale. Thus, in order to be varying during the last Hubble time but not drastically, $\phi$ has to be very light with typical mass $m \sim H_0 \sim 10^{-33}$ eV. As in the case of quintessence, this induces difficulties to understand how such a light mass is stable against radiative corrections.

There are a lot of theoretical motivations to point toward a deviation from Einstein gravity on large scales (such as the cosmological constant problem or string phenomenology). I have presented two independent ways of testing these ideas, the test of the inverse square law on astrophysical scales using weak gravitational lensing and the test of Einstein equivalence principle using the study of the variation of the constants. This may shed some new light on the law of gravity and it offers a new links between astrophysics, cosmology, and high-energy physics complementary to the existing ones offered by primordial cosmology.

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